**In the name of God**

**The second quiz.** **Linear model**

1- Let $\mathbf{Y} \sim N_n(\mathbf{0},\mathbf{I}_n)$ and let $\mathbf{A}$ be a symmetric matrix. Then $\mathbf{Y}^T\mathbf{A}\mathbf{Y} \sim \chi^2_{(r)}$ if and only if $\mathbf{A}$ is idempotent of rank r.

2- Suppose that $\mathbf{Y} \sim N_n(\boldsymbol{\mu},\boldsymbol{\Sigma})$, where $\boldsymbol{\mu}=\mu\mathbf{1}$ and $Var(Y_i)=\sigma^2$ and $Cov(Y_i,Y_j)=\rho\sigma^2$ for i=j. Show that $(n-1)S^2 / [\sigma^2(1-\rho)] \sim \chi^2_{(n-1)}$

3. Let $X_1, X_2, \cdots, X_n$ be random variables with a common mean $\mu$. Suppose that $cov(X_i, X_j) = 0$ for all $i$ and $j$ such that $j > i+1$. If

$$Q_1 = \sum_{i=1}^{n} (X_i - \bar{X})^2$$

and

$$Q_2 = (X_1 - X_2)^2 + (X_2 - X_3)^2 + \cdots + (X_{n-1} - X_n)^2 + (X_n - X_1)^2,$$

prove that

$$E\left[\frac{3Q_1 - Q_2}{n(n-3)}\right] = var(\bar{X})$$

Good luck.